Numeracy Essentials

Section 1 – Number Skills

Reading and writing numbers

All numbers should be written correctly. Most pupils are able to read, write and say numbers up to a thousand, but often have difficulty with larger numbers. It is common practice to use spaces between each group of three figures. eg. 34 000. Apostrophes should be discouraged e.g. 34’000.

Decimals should be spoken digit by digit eg 0.34 is “Zero (or nought) point three four” (NOT thirty four).

Money should always be written to two decimal places. Please do not accept £7.2 for example. There should only be one money sign (£ or p) but NOT both. E.g. £7.20 or 720p but never £7.20p

Order of Operations

It is important that pupils follow the correct order of operations for arithmetic calculations. Most will be familiar with the mnemonic: **BIDMAS.**

Brackets, **I**ndex (eg squared, square roots etc), **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction

This shows the order in which calculations should be completed. eg

5 + 3 x 4 means	**<u>NOT</u>**	5 + 3 x 4
5 + 12		8 x 4
= <u>17</u> ✓		= <u>32</u> **X**

Brackets are done first, then the **I**ndices, **M**ultiplication and **D**ivision and finally, **A**ddition and **S**ubtraction. Some students may recall this as BODMAS, where the O stands for “Order”, which also means a power / index.

Calculators

Some pupils are over-dependent on the use of calculators for simple calculations. Wherever possible pupils should be encouraged to use mental or pencil and paper methods. It is, however, necessary to give consideration to the ability of the pupil and the objectives of the task in hand. In order to complete a task successfully it may be necessary for pupils to use a calculator for what is perceived to be a relatively simple calculation. This should be allowed if progress within the subject area is to be made. Before completing the calculation pupils should be encouraged to make an estimate of the answer. Having completed the calculation on the calculator they should consider whether the answer is reasonable in the context of the question. Please do not allow students to use their mobile phones as calculators. All students should have a basic calculator as part of their essential equipment.

Mental Calculations

Most pupils should be able to carry out the following processes mentally though the speed with which they do it will vary considerably.

- recall addition and subtraction facts up to 20
- recall multiplication and division facts for tables up to 10 x 10.

Written Calculations

Pupils often use the ' = ' sign incorrectly. When doing a series of operations they sometimes write mathematical sentences which are untrue.

eg 5 x 4 = 20 + 3 = 23 – 8 = 15 since 5 x 4 ≠ 15

It is important that all teachers encourage pupils to write such calculations correctly.

eg 5 x 4 = 20
20 + 3 = 23
23 – 8 = 15 ✓

> Calculations should always be set out one line underneath the next like this.

The ' = ' sign should only be used when both sides of an operation have the same value.
The '≈' (approximately equal to) sign should be used when estimating answers.

eg 2 378 – 412 ≈ 2 400 – 400
2 400 – 400 = 2 000 ✓

Pencil & Paper Calculations

Before completing any calculation, pupils should be encouraged to estimate a rough value for what they expect the answer to be. This should be done by rounding the numbers and mentally calculating the approximate answer. After completing the calculation they should be asked to consider whether or not their answer is reasonable in the context of the question.

There is no necessity to use a particular method for any of these calculations and any with which the pupil is familiar and confident should be used. The following methods are some with which pupils may be familiar.

Addition & Subtraction

Addition 3 456 + 975

3 456
\+ 975
4 431
1 1 1

> Remember to estimate the answer:
>
> 3 500 + 1 000 = 4 500

Subtraction 8003 - 2569

Estimate: 8 000 – 3 000 = 5000

eg
7 9 9 1
~~8~~ ~~0~~ ~~0~~ 3
-2 5 6 9
5 4 3 4

> Sometimes it will be necessary to borrow over one or more digits.

Subtraction by 'counting on'

Estimate: 8 000 – 3 000 = 5 000

eg 8 003 – 2 569

Start	Add
2 569	1
2 570	30
2 600	400
3 000	5 000
8 000	3
Total	5 434

Multiplication and Division by 10,100,1000 etc.

When a number is multiplied by 10 its value has increased tenfold and each digit will move one place to the left. When multiplying by 100 each digit moves two places to the left, and so on… Any empty columns will be filled with zeros so that place value is maintained when the numbers are written without column headings. The same method is used for decimals.

eg. 46 x 100 = 4 600

Th	H	T	U
		4	6
4	6	0	0

eg. 5.34 x 10 = 53.4

H	T	U	.	t	h
		5	.	3	4
	5	3	.	4	

Empty spaces after the decimal point are not filled with zeros. The place value of the numbers is unaffected by these spaces.

When dividing by 10 each digit is moved one place to the right so making it smaller.

eg. 350 ÷ 10 = 35

H	T	U	.	t	H
3	5	0	.		
	3	5	.		

eg. 53÷ 100 = 0.534

H	T	U	.	t	H
	5	3	.		
		0	.	5	3

When the calculation results in a decimal, the units column must be filled with a zero to maintain the place value of the numbers.

Multiplication

$$\begin{array}{r} 3\,2\,7 \\ \times \quad 5\,3 \\ \hline 9\,8_2\,1 \\ 1\,6_1 3_3\,5\,0 \\ \hline 1\,7\,3\,3\,1 \\ \hline \end{array}$$

9 8₂ 1 ← 327 x 3

1 6₁3₃ 5 0 ← 327 x 50

Remember to include the 0 in the second row because we move into the tens column

Conventional multiplication as set out above may not suit all pupils and teachers should be aware that other methods may be employed by some pupils.

eg 327 x 53 Estimate: 300 x 50 = 15 000

X	300	20	7	Total
50	15 000	1000	350	16 350
3	900	60	21	981
Total	15900	1060	371	17331

Division

There is no need for students to be able to use long division in Mathematics lessons or any other subject areas. Short division is sufficient for non-calculator methods.

e.g. 4 707 ÷ 3

$$\begin{array}{r|l} & 1\;5\;6\;9 \\ \hline 3 & 4\;{}^{1}7\;{}^{2}0\;{}^{2}7 \end{array}$$

Any remainders in this type of calculation should be written as a fraction or decimal by dividing the remainder by the number by which the calculation has been divided.

Short division can also be used for larger numbers.

e.g. 628 935 ÷ 25

$$\begin{array}{r|l} & 0\;\;2\;\;5\;\;1\;\;5\;\;7\;\;r\,10 \\ \hline 25 & 6\;\;{}^{6}2\;\;{}^{12}8\;\;{}^{3}9\;\;{}^{14}3\;\;{}^{18}5 \end{array}$$

> It is useful to write out the multiples of the number you are dividing by, as a point of reference throughout the question.
>
> 25, 50, 75, 100, 125, 150, 175, 200 …

So the answer to 628 935 ÷ 25 = 25 157 $\frac{10}{25}$

Which can be simplified to 25 157 $\frac{2}{5}$ by cancelling the fraction.

Multiplying Decimals

- As always, estimate the answer.
- Complete the calculation as if there were no decimal points.
- In the answer insert a decimal point so that there are the same number of decimal places in the answer as there were in the original question.
- Check to see if the answer is reasonable

eg (i) 1.2 x 0.3 $\approx$ 1 x 0.3 = 0.3

Ignoring the decimal points, this will be calculated as 12 x 3 = 36 and will now need <u>two</u> decimal places in the answer.

1.2 x 0.3 = 0.36

Similarly:

eg (ii) 43.14 x 3.5 $\approx$ 40 x 4 = 160

	4	3	.	1	4		(2 decimal places)
x			3	.	5		(1 decimal place)
	2	1	5	7	0		
1	2	9	4	2	0		
1	5	0 .	9	9	0		(3 dp needed in the answer)

Percentages

Calculating Percentages of a quantity

Methods for calculating percentages of a quantity vary depending upon the percentage required. Pupils should be aware that fractions, decimals and percentages are different ways of representing part of a whole and know the simple equivalents

eg 10% = $^{1}/_{10}$ 12% = 0.12

Where percentages have simple fraction equivalents, fractions of the amount can be calculated.

eg. i) To find 50% of an amount, halve the amount.
  ii)To find 75% of an amount, find a quarter by dividing by four and then multiply it by three.

Most other percentages can be found by finding 10%, by dividing by 10, and then finding multiples of that amount

eg. To find 30% of an amount first find 10% by dividing the amount by 10 and then multiply this by three.
  30% = 3x10%

  Similarly: 5% = half of 10% and 15% = 10% + 5% and 1% = 10% ÷ 10

Other percentages can be calculated in this way.

When using the calculator it is usual to think of the percentage as a decimal. Pupils should be encouraged to convert the question to a sentence containing mathematical symbols. ('of' means X)

  eg. Find 27% of £350 becomes 0.27 X £350

and this is how it should be entered into the calculator.

Other methods involve thinking of a percentage as a part of 100 so the calculation becomes

  350 ÷ 100 x 27
  = 94.5

> Money must always be written to 2 decimal places!

However, given that the calculation involves money we must always remember to round it to 2 decimal places and use the pound sign. So 94.5 (on the calculator) becomes an actual answer of £94.50

Calculating the amount as a percentage

In every case the amount should be expressed as a fraction of the original amount and then converted to a percentage in one of the following ways:

> This is commonly used when working out a test score as a percentage

What is 39 as a percentage of 64?
(Using a calculator)

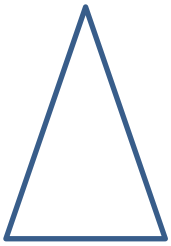
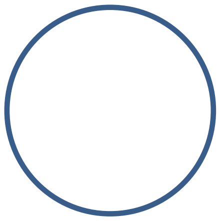
$\frac{39}{64}$ = 39 ÷ 64 = 0.609375 = 60.9% to 1 decimal place

Section 2 – Measuring Skills

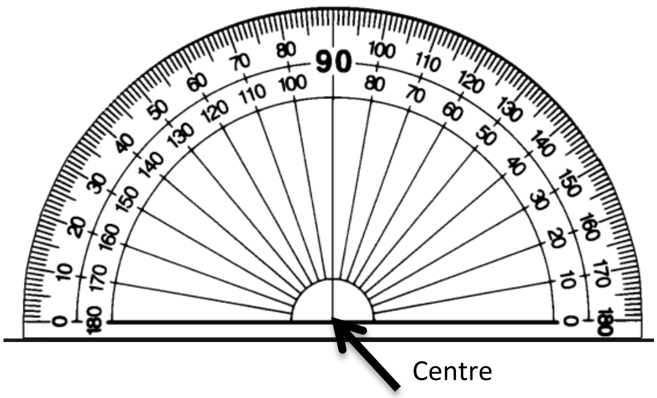
Area and Perimeter

Area is the space inside a shape (this usually involves multiplying). It is measured in units squared (e.g. cm^2)

Perimeter is the distance around the outside of a shape (this usually involves adding). It is measured in units (e.g. cm)

Rectangle	Triangle	Circle
		
Area = Base x Height (or length x width)	Area = $\frac{\text{Base x Height*}}{2}$ Must use the perpendicular height NOT the length of the side of the triangle.	Area = π x $radius^2$ Circumference = 2 x π x radius

Using a protractor



Students often confuse how to use a protractor to measure angles.

The centre of the protractor (upside down T) should be placed on the corner of the angle. The zero line of the protractor needs to be lined up with one side of the angle.
You read the set of numbers from your zero line.

Students should always check the sensibility of their angle (i.e. should it be less than or greater than 90 degrees.

Metric and Imperial units

Students should be familiar with using both metric and imperial units. They should know the metric conversions and rough imperial equivalents.

Bearings

Bearings are used to state a direction relative to another point of interest (e.g. a ship). They should be given in 3 figures and always go clockwise from north. For example, the bearing of due East would be recorded as 090˚

Section 3 – Data Skills

It is important that graphs and diagrams are drawn on the appropriate paper:

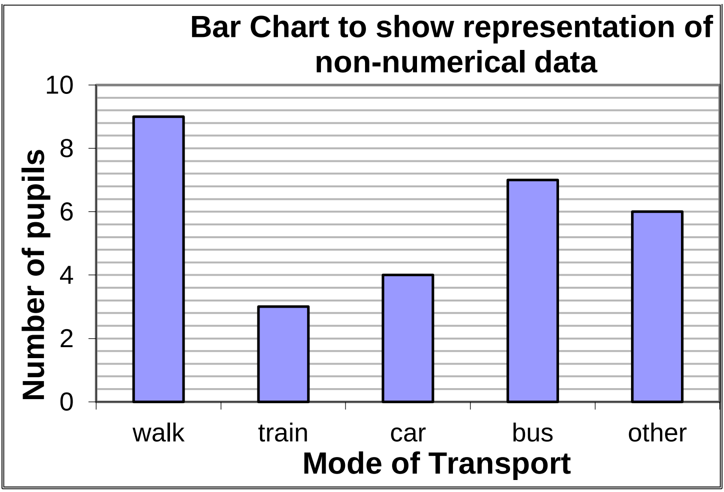
- Bar charts, line graphs and scatter graphs on squared or graph paper
- Pie charts on plain paper (though can be done in books)

Bar Charts

These are the diagrams most frequently used in areas of the curriculum other than mathematics. The way in which the graph is drawn depends on the type of data to be processed.

Graphs should be drawn with **gaps between the bars** if the data categories are not numerical (colours, makes of car, subject preference, etc). There should also be gaps if the data is numeric but can only take a particular value (shoe size, KS3 level, etc). In cases where there are gaps in the graph the horizontal axis will be labelled beneath the columns. **Almost all** bar charts should contain gaps between the bars!

eg.



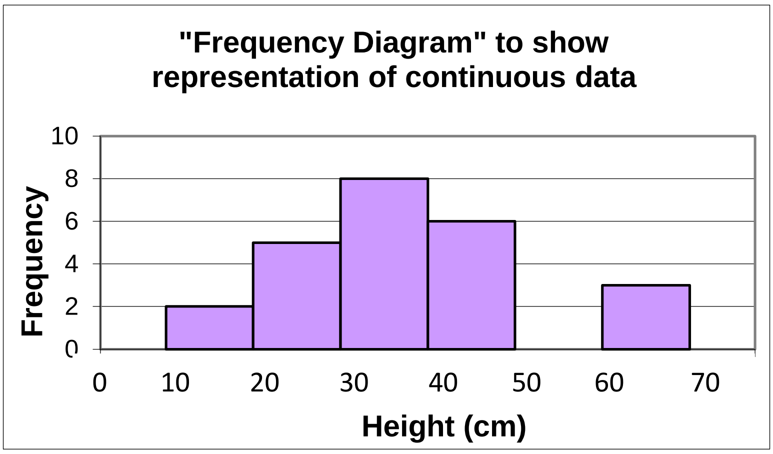
Grouped Frequency Diagrams

Where data is continuous, eg. lengths, we display this information in a frequency diagram. This is similar to a bar chart but does not contain gaps due to the nature of the data. The horizontal scale should be like the scale used for a graph on which points are plotted. When interpreting a frequency diagram we cannot read exact values as data is grouped into categories. Frequency diagrams can also be known as Histograms.

As this is grouped data, we need to use inequalities to represent the groupings (otherwise 20, for example, could appear in two groups)

The ≤ symbol shows that the value is included. So this group would include 10 and everything up to (but not including) 20.

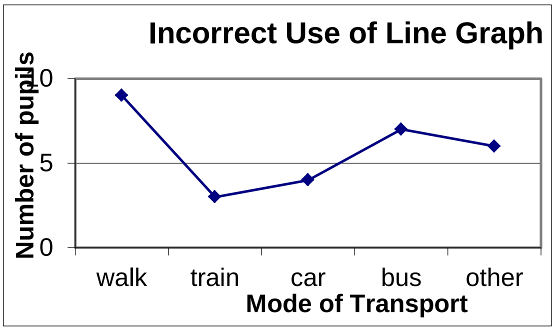
Height	Frequency
$0 \le h < 10$	0
$10 \le h < 20$	2
$20 \le h < 30$	5
$30 \le h < 40$	8
$40 \le h < 50$	6
$50 \le h < 60$	0
$60 \le h < 710$	3



Line Graphs

Line graphs should only be used with data in which the order in which the categories are written is significant (this will be time related in most cases).

Points are joined if the graph shows a trend or when the data values between the plotted points make sense to be included.



The above shows an **incorrect** use of a line graph as there is no order along the bottom. A bar chart would be a more suitable way to represent the data.

Computer Drawn Graphs & Diagrams

Pupils may use computerised graphs, though they must be careful not to produce them unless they are relevant. All graphs should be commented on and explained. They should also ensure that they are correctly labelled.

Pie Charts

Pie charts are a good way of displaying data visually and show the different proportions clearly. Students often have the misconception that pie charts show percentages and, whilst this can be true, it is not the main purpose of drawing the pie chart. In mathematics lessons it is extremely rare to encounter pie charts and percentages together.

Example

The following table shows the results of a survey of 30 pupils travelling to school. Show this on a pie chart.

Pupils should first work out the share of 360° to be allocated to **one** item and then multiply this by its frequency.

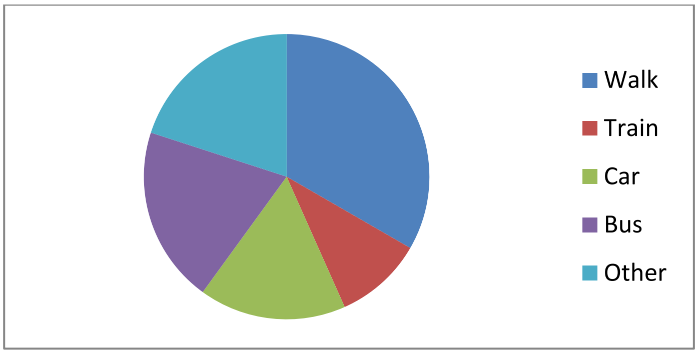
Each pupil has $360 \div 30 = 12^\circ$ of the pie chart.

We then need to multiply each category by this number.

Mode of Transport	Frequency	Angle Calculation (frequency x angle)
Walk	10	$10 \times 12 = 120^\circ$
Train	3	$3 \times 12 = 36^\circ$
Car	5	$5 \times 12 = 60^\circ$
Bus	6	$6 \times 12 = 72^\circ$
Other	6	$6 \times 12 = 72^\circ$
Total	**30**	**360°**

It is important that we check the angles add up to 360°

A protractor and a compass is then needed to accurately construct the pie chart.



It is common practice to colour code a pie chart. It is often better to colour each section and make a key next to it rather than try to fit the category names into each sector.

Sometimes 360 ÷ the frequency gives a long decimal number. Any calculations of angles should be rounded to the nearest degree only at the **final stage of the calculation.** If the number of items to be shown is 47 each item will need:

$$360 \div 47 = 7.659574468^\circ$$

This complete number should be used when multiplying by the frequency and then rounded to the nearest degree. If we round too soon it can mean that our angles won't add up to 360^0

Pie charts are a good way of representing proportions visually. They allow you to compare two data sets, but be mindful that they do not show actual quantities.

A pie chart is best used when there are 3 – 8 categories of data. Any more than this can it difficult to read.

Averages

Three different averages are commonly used:

Mean – is calculated by adding up all the values and dividing by the number of values.

Median – is the middle value when a set of values has been arranged in ascending order.

Mode - is the most common value. It is sometimes called the modal group.

eg. for the following values: 3, 2, 5, 8, 4, 3, 6, 3, 2,

Mean = $\frac{3 + 2 + 5 + 8 + 4 + 3 + 6 + 3 + 2}{9} = \frac{36}{9} = 4$

Median – is 3 because 3 is in the middle when the values are put in order.

2, 2, 3, 3, (3), 4, 5, 6, 8

Mode – is 3 because 3 is the value which occurs most often.

Range

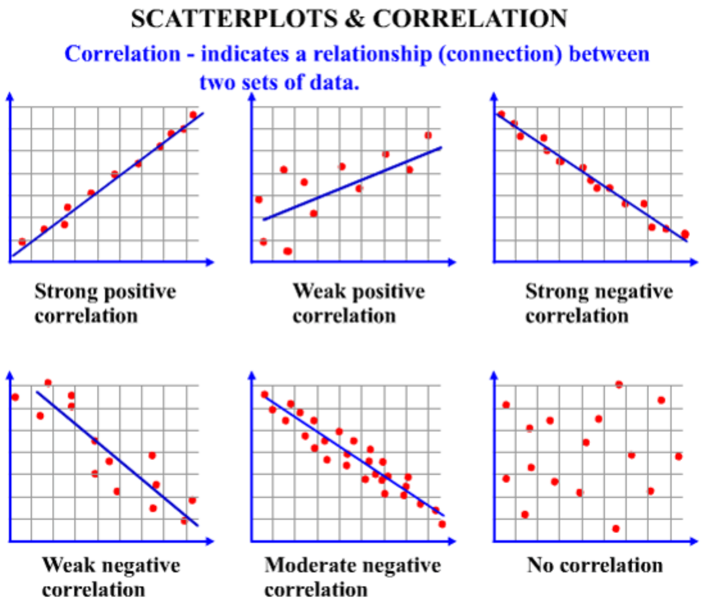
The range of a set of data is the difference between the highest and the lowest data values. It is not an average but is often used along side them to measure the spread of the data.

eg. If in an examination the highest mark is 80% and the lowest mark is 45%, the range is 35% because 80% - 45% = 35%

The range is always a **single number** , so it is **NOT** e.g**.** 45% - 80%

Scatter graphs

These are used to compare two sets of numerical data. The two values are plotted on two axes labelled as for continuous data. Sometimes there may be a correlation between the two variables, if so a 'line of best fit' should be drawn. Lines of best fit should always be straight and drawn with a ruler. It need not pass through the origin.



The degree of correlation between the two sets of data is determined by the proximity of the points to the 'line of best fit'. We describe the correlation as strong (where the points are close to the line of best fit) or weak (where the points are scattered less uniformly).

Questionnaires

Questionnaires should:

- Have a time frame
- Have at least 3 response boxes
- Include all possibilities
- Never contain vague or overlapping options.

e.g. How much money do you spend on lunch each week?

☐ Less than £5 ☐ £5 - £7.50 ☐ £7.51 - £10.00 ☐ More than £10

Data Collection Sheets

A data collection sheet is a quick and easy way of collecting data. It is like a blank tally chart and is filled in when asking questionnaires. E.g.

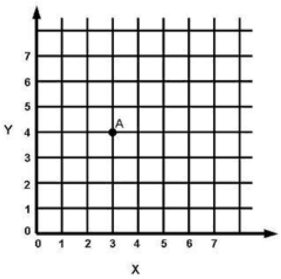
Lunch Money	Tally	Frequency
Less than £5		
£5 - £7.50		
£7.51 - £10.00		
More than £10		

Tally is filled in using sticks (5 makes a gate) 卌 Frequency is the tally written as a number.

Section 4 – Algebra

Plotting Points

When drawing a diagram on which points have to be plotted some pupils will need to be reminded that the numbers written on the axes must be on the lines not in the spaces. The co-ordinates should be plotted on the cross where two lines meet, rather than in the middle of a square.



The co-ordinates (3, 4) are plotted 3 across, 4 up.

Axes

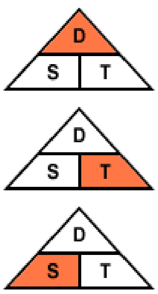
When drawing graphs to represent experimental data it is usual to use the horizontal (x) axis for the variable which has a regular class interval.

eg In an experiment in which temperature is taken every 5 minutes the horizontal axis would be used for time and the vertical axis for temperature.

Having plotted points pupils can sometimes be confused as to whether or not they should join the points. If the results are from an experiment then a 'line of best fit' will usually be needed. Further details appear in the following section on Data Handling.

Formulae

One of the most common areas of cross curricular algebra is the use of formulae.



Distance = Speed x Time

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

Formula pyramids are a good way of helping to rearrange a formula depending on what information you have. You highlight (or cover) what you're looking for. What's left is the calculation that needs to be done.

A harder example of a formula being used is $s = ut + \frac{1}{2}at^2$ In this case, it is important that the BIDMAS rules are adhered to.

E.g. if u = 5, t = 10 and a = 25

$S = (5 \times 10) + \frac{1}{2}(25 \times (10^2))$

$= 50 + \frac{1}{2}(2500)$

$= 50 + 1250$

$= 1300$

When writing letters on their own, there is no need for the number 1 to appear. Eg we write x not 1x.

Purpose

Bar Chart	A bar chart can be used to represent discrete data (which is often worded). It is a visual way of representing a data set and can clearly show the mode. It allows us to easily compare different data, summarise a large data set in visual form, and clarify trends more easily than tables. On their own though, bar charts don't really tell us a great deal and are often done for the sake of doing a graph.
Frequency diagram	A frequency diagram represents continuous data (numerical data with no gaps). It has the same purpose as a bar chart but instead displays grouped data.
Pie Chart	A pie chart is a good way of looking at the proportions of data. If two data sets are used then a comparison can be made more easily (especially if the data sets contain different frequencies) than from the numerical data alone. They show proportions though and so whilst one sector may be bigger on one pie chart than another, it doesn't mean that it has a higher frequency than the other.
Scatter Graph	A scatter graph allows us to see if there is a correlation between two data sets. The stronger the correlation, the tighter together the points will be, allowing a line of best fit to be drawn.
Line Graph	A line graph usually shows the trend of data over time. It would be drawn to see if long term patterns exist.
Averages	Averages are used as a measure of what is typical of a data set. The mean tends to be the average used the most but can easily be affected by extreme pieces of data. The median will avoid this problem but doesn't take all the data into consideration.
Questionnaire	A questionnaire should be used to collect data first hand when it is not appropriate to use secondary data.